

# Obstacles to extending $R$ -parity violation to Supersymmetric SU(5)

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## Abstract

We explore the consequences of promoting bilinear  $R$ -parity violation, usually formulated in the minimal supersymmetric standard model framework, to a supersymmetric SU(5) grand unified theory. We observe that the limits on proton decay and neutrino mass place tight constraints on the bilinear SU(5)  $R$ -parity violating parameters creating a different doublet-triplet issue which cannot be resolved by an extension of the usual fine-tuning in the symmetry breaking scalar sector. If the parameters are made to satisfy the constraints, albeit unnaturally, there remains no room for the possibility to correct the SU(5) fermion mass ratios by introducing  $R$ -parity violation.

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## I. INTRODUCTION

In the standard model (SM), the Yukawa couplings are hand-picked to explain the observed fermion masses via the Higgs mechanism. In the minimal supersymmetric standard model (MSSM), fermion masses are obtained from two Higgs fields  $H_u$  – which gives mass to the up-type quarks – and  $H_d$  – which is responsible for the masses of the down-type quarks and charged leptons. Due to the absence of the right-handed fields the neutrinos cannot acquire a Dirac mass. Further, if lepton number conservation is imposed then this forbids a neutrino Majorana mass.

When this model is embedded in a grand unified theory (GUT), such as SU(5), the unification of couplings is an attractive consequence. The SU(5) symmetry also requires the ratio of down-type quark and charged lepton masses of each generation to be unity at the GUT scale.

$$\left(\frac{m_d}{m_\ell}\right)_i = 1 \quad i = 1, 2, 3, \quad (1.1)$$

while the ratios obtained by extrapolating the measured masses are

$$\frac{m_d}{m_e} \sim 2.6, \quad \frac{m_s}{m_\mu} \sim 0.23, \quad \frac{m_b}{m_\tau} \sim 0.81. \quad (1.2)$$

If the requirement of  $R$ -parity symmetry is relaxed one of the main motivations of MSSM – the LSP dark matter candidate – is lost. But  $R$ -parity violation (RPV) also has its virtues. It can be used to explain the observed pattern of neutrino masses and mixings [1, 2]. Hence it is pertinent to ask the question whether RPV can address the mismatch of wrong fermion mass ratios posed in supersymmetric (SUSY) SU(5) [3]. The issue can be alleviated using the trilinear  $A$  terms [4]. Alternatively, a solution can be obtained by adding  $5 + \bar{5}$  vector-like matter fields in SUSY SU(5) [5]. For other approaches using non-minimal models see Ref. [6].

Non-observation of proton decay, e.g., at SuperKamiokande [7, 8], poses severe constraints [9] on grand unified theories. In non-SUSY theories, typified by SU(5), proton decay is driven by dim-6 operators. In  $R$ -parity conserving SUSY SU(5), the existence of sfermions at the electroweak scale allows proton decay to proceed through dim-5 operators [10–12]. However, when RPV is admitted proton decay can arise even from dim-4 operators, which puts severe restrictions on the size of  $R$ -parity violating parameters [13–15].

Neutrino mass is another area where RPV interactions which violate lepton number can play an important role. RPV results in mixing between neutralinos and neutrinos. This leads to one neutrino state becoming massive [1, 2]. The observed smallness of neutrino masses limit the size of  $R$ -parity violating interactions.

In this paper we show that extension of bilinear  $R$ -parity violation of MSSM to the SU(5) theory faces a serious obstacle in maintaining consistency with proton decay and neutrino mass constraints. Further it is *not* possible to find a satisfactory resolution of the issue of wrong fermion mass ratios within SUSY SU(5) even in the context of  $R$ -parity violation unless severe accidental fine-tunings amongst various uncorrelated sectors are entertained [3].

## II. RPV SUSY SU(5): A FLASHBACK

In minimal SUSY SU(5) the matter fields – all left-handed – are contained in

$$\bar{5}_i \equiv \underbrace{(\bar{3}, 1)}_{d_{0i}^c} + \underbrace{(1, 2)}_{L_{0i}} \quad \text{and} \quad 10_i \equiv \underbrace{(\bar{3}, 1)}_{u_{0i}^c} + \underbrace{(3, 2)}_{Q_{0i}} + \underbrace{(1, 1)}_{e_{0i}^c}, \quad (2.1)$$

where  $i = 1, 2, 3$  is the generation index and the numbers in the parentheses are the SU(3)<sub>c</sub> and SU(2)<sub>L</sub> quantum numbers.  $L_1 = (\nu_e, e)_L$  and  $Q_1 = (u, d)_L$  stand for left superfields.  $d^c \equiv (d_R)^C$ , where  $d_R$  is the right chiral down quark superfield. The same is true for the other right superfields  $u_R$  and  $e_R$ . The subscript 0 is indicative of the flavour basis. Colour indices are suppressed. We express the above in the form:

$$\bar{5}_i = \begin{pmatrix} d_0^c \\ \epsilon_2 L_0 \end{pmatrix}_i \quad 10_i = \begin{pmatrix} \epsilon_3 u_0^c & Q_0 \\ -Q_0^T & \epsilon_2 e_0^c \end{pmatrix}_i. \quad (2.2)$$

$\epsilon_n$  represents the  $n$ -dimensional completely antisymmetric tensor with  $\epsilon_{12} = +1$  and  $\epsilon_{123} = +1$ .

The Higgs fields are contained in

$$5_H = \begin{pmatrix} T \\ H_u \end{pmatrix} \quad \bar{5}_H = \begin{pmatrix} \bar{T} \\ H_d \end{pmatrix}. \quad (2.3)$$

The scalar field which breaks SU(5) to the SM resides in a 24-plet adjoint representation.  $T, \bar{T}$  represent colour triplets having masses around the GUT scale,  $M_{GUT} \sim 10^{16}$  GeV.

Once  $R$ -parity violation is considered, there is no quantum number that distinguishes  $\bar{5}_i$  from  $\bar{5}_H$  and as a result it is convenient to club these using the following notation

$$\bar{5}_\alpha = \begin{pmatrix} \bar{3}_\alpha \\ \bar{2}_\alpha \end{pmatrix} \quad \alpha = 0, 1, 2, 3, \quad (2.4)$$

with  $\bar{5}_0 = \bar{5}_H$ . We can then write the superpotential keeping only the relevant terms for this discussion as<sup>1</sup>

$$W \in \bar{5}_\alpha (M_\alpha + \eta_\alpha 24) 5_H + \frac{1}{2} Y_{\alpha\beta k}^5 \bar{5}_\alpha \bar{5}_\beta 10_k + Y_{ij}^{10} 10_i 10_j 5_H. \quad (2.5)$$

$Y_{\alpha\beta k}^5$  is antisymmetric in the first two indices while  $Y_{ij}^{10}$  is symmetric under  $i \leftrightarrow j$ . Yukawa couplings for  $H_u$  and  $H_d$  are obtained from  $Y_{ij}^{10}$  and  $Y_{0jk}^5$  respectively. We choose the fermion basis states so that the latter is diagonal, i.e.,  $Y_{0jk}^5 = Y_j^5 \delta_{jk}$ .  $Y_{ijk}^5$  are trilinear RPV couplings which we take to be absent, the entire  $R$ -parity violation arising from the bilinear mixing encoded in the first term in Eqn. (2.5). We remark later about the possibility of keeping  $Y_{ijk}^5$  non-zero and fine-tuning them to cancel off the effects arising from the bilinear  $R$ -parity violation.

$M_\alpha$  represent SU(5)-invariant mass terms and their natural scale is  $\mathcal{O}(M_{GUT})$ . The coupling  $\eta$  is expected to be  $\mathcal{O}(1)$  or smaller.

The mass terms of the superpotential are given by

$$\bar{3}_\alpha \mathcal{M}_\alpha T + \bar{2}_\alpha \mu_\alpha H_u, \quad (2.6)$$

with

$$\mathcal{M}_\alpha = M_\alpha + 2\eta_\alpha V, \quad (2.7)$$

$$\mu_\alpha = M_\alpha - 3\eta_\alpha V, \quad (2.8)$$

where  $V$  is the vacuum expectation value (VEV) received by the 24-plet scalar field around the GUT scale.  $\mathcal{M}_i$  ( $\mu_i$ ) stand for bilinear RPV couplings involving colour triplets (SU(2) doublets). In SUSY with RPV neutrinos (sneutrinos) mix with neutralinos (neutral Higgs) and charged leptons (sleptons) mix with charginos (charged Higgs) due to the presence of

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<sup>1</sup> The superpotential has the matter fields in the flavour basis. We have suppressed the subscript 0 to avoid cluttering of the notation.

$\mu_i$ . When RPV SUSY is embedded in an SU(5) GUT the additional bilinear RPV couplings  $\mathcal{M}_i$  allow the fermionic members of the colour triplet superfields  $T$  and  $\bar{T}$  (scalars of  $T$  and  $\bar{T}$ ) to mix with down-type quarks (squarks). We will mainly focus on the phenomenology of these new RPV couplings in SUSY SU(5).

So long as SU(5) is exact, both  $\mathcal{M}_\alpha$  and  $\mu_\alpha$  are forced to lie at the same scale as  $M_\alpha$ . When SU(5) breaking occurs, the 24-plet acquires a GUT-scale VEV. The challenge to keep  $\mathcal{M}_0$  at the GUT level while maintaining  $\mu_0$  at the electroweak scale is known as the doublet-triplet splitting problem and it calls for a large fine-tuning of the  $R$ -parity conserving term. Specifically,  $\eta_0$  can be fine-tuned such that a cancellation occurs in Eqn. (2.8) between the two  $\mathcal{O}(M_{GUT})$  terms leaving a tiny  $\mu_0 \sim \mathcal{O}(M_W)$  while from Eqn. (2.7)  $\mathcal{M}_0$  remains at the GUT scale. The same equations appear to leave open the option to similarly fine-tune the bilinear RPV-terms  $\mathcal{M}_i$  and  $\mu_i$  through the  $\eta_i$ . However, it is clear from the difference in sign in the two equations that if one of  $\mathcal{M}_i$  and  $\mu_i$  is fine-tuned to a small value the other must remain at the GUT scale. In this note we stress that *both*  $\mathcal{M}_i$  and  $\mu_i$  are required to be significantly smaller than  $M_{GUT}$  from the limits on proton decay and from the neutrino mass scale, respectively, which cannot be realised in the above manner. Thus an extension of bilinear  $R$ -parity violation to SU(5) is fraught with an inherent hurdle.

The Lagrangian containing the mass terms for the colour triplet fermions including the contribution from the first term in Eqn. (2.6) can be schematically written as<sup>2</sup>:

$$\begin{pmatrix} \bar{T}_0 & d_{0i}^c \end{pmatrix} \begin{pmatrix} \mathcal{M}_0 & 0 \\ \mathcal{M}_i & m^{\text{diag}} \end{pmatrix} \begin{pmatrix} T_0 \\ d_{0j} \end{pmatrix}, \quad (2.9)$$

where, due to the choice of the  $d$ -type quark basis,  $(m^{\text{diag}})_{ij} = \delta_{ij} Y_j^5 v_d$  is a  $3 \times 3$  diagonal matrix ( $v_d$  being the vacuum expectation value of  $H_d$ ). The mass matrix in (2.9) can be diagonalised by a bi-unitary transformation (see Appendix A):

$$\begin{pmatrix} \bar{T} \\ d_i^c \end{pmatrix} = \begin{pmatrix} U_{00} & U_{0j} \\ U_{i0} & U_{ij} \end{pmatrix} \begin{pmatrix} \bar{T}_0 \\ d_{0j}^c \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} T \\ d_i \end{pmatrix} = \begin{pmatrix} V_{00} & V_{0j} \\ V_{i0} & V_{ij} \end{pmatrix} \begin{pmatrix} T_0 \\ d_{0j} \end{pmatrix}. \quad (2.10)$$

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<sup>2</sup> A typical fermion mass term can be expressed as  $m_{12} \{\psi_{f_1}^C\}^T \mathcal{C}^{-1} \psi_{f_2}$  where  $\mathcal{C}$  is the charge conjugation operator. For chiral fermions this is  $m_{12} \{\psi_{f_1}^C\}^T P_L \mathcal{C}^{-1} P_L \psi_{f_2} + \text{h.c.}$  which is written here in condensed form as  $m_{12} \{f_1^c\}_L \{f_2\}_L$ .

In the mass basis Eqn. (2.9) becomes:

$$\begin{pmatrix} \bar{T} & d_i^c \end{pmatrix} \begin{pmatrix} M_T & 0 \\ 0 & m_d \end{pmatrix} \begin{pmatrix} T \\ d_i \end{pmatrix}, \quad (2.11)$$

with  $m_d = \text{diag}(m_{d_i})$ ,  $m_{d_i}$  being the down-type quark masses at the grand unification scale.

In Eqn. (2.10)  $V$  is close to the identity matrix, as illustrated in the one generation case in Appendix A.

The matrix  $U$  is then given by [3, 5]

$$U_{00} = \frac{1}{\sqrt{1 + |x_i|^2}}, \quad (2.12)$$

$$U_{0i} = -U_{i0} = \frac{x_i}{\sqrt{1 + |x_i|^2}}, \quad (2.13)$$

$$U_{ij} = \delta_{ij} - \frac{x_i x_j}{\sqrt{1 + |x_i|^2} (1 + \sqrt{1 + |x_i|^2})}, \quad (2.14)$$

where  $x_i = \mathcal{M}_i/\mathcal{M}_0$  and  $|x_i|^2 \equiv \sum_i x_i^2$ .

So far we have focussed on the first term of Eqn. (2.6) which mixes SU(2) singlet, colour anti-triplets,  $\bar{T}$ , and the  $d_i^c$  states. The second term of the equation produces the commonly considered bilinear  $R$ -parity violating mixing between the  $H_d$  and the  $L_i$  which are colour singlet, SU(2) doublets. The upshot of this is mixing between the charged leptons and the chargino on the one hand and the neutrinos and the neutralino on the other.

For illustration, the chargino mass matrix is extended to incorporate mixing of the charged leptons with the superpartner of the Higgs  $H_d$ . Denoting by  $M_2$  the SU(2) gaugino mass, the extended mass matrix can be written as

$$\begin{pmatrix} \widetilde{W}_0^- & \widetilde{H}_{0d}^- & e_{0i} \end{pmatrix} \begin{pmatrix} M_2 & gv_u/\sqrt{2} & 0 \\ gv_d/\sqrt{2} & \mu_0 & 0 \\ 0 & \mu_i & m^{\text{diag}} \end{pmatrix} \begin{pmatrix} \widetilde{W}_0^+ \\ \widetilde{H}_{0u}^+ \\ e_{0j}^c \end{pmatrix}. \quad (2.15)$$

It is important to note at this point that the same  $m^{\text{diag}}$  appears here as in Eqn. (2.9) due to the SU(5) symmetry which in turn predicts wrong fermion mass relations  $m_{d_i} = m_{\ell_i}$  in  $R$ -parity conserving SU(5) supersymmetry. We will explore whether due to the presence of RPV bilinear couplings  $\mathcal{M}_i$  and  $\mu_i$ , the ratios correct themselves.

Experimental evidences accumulating at the LHC exclude gluino masses of a TeV or less. If  $M_2$  is also greater than 1 TeV then the wino-state essentially decouples in Eqn. (2.15).

Accordingly, let us concentrate on the following submatrix in Eqn. (2.15) which has a similar structure as the mass matrix in Eqn. (2.9):

$$\begin{pmatrix} \tilde{H}_{0d}^- & e_{0i} \end{pmatrix} \begin{pmatrix} \mu_0 & 0 \\ \mu_i & m^{\text{diag}} \end{pmatrix} \begin{pmatrix} \tilde{H}_{0u}^+ \\ e_{0j}^c \end{pmatrix}. \quad (2.16)$$

Here  $\mu_0$  is at the electroweak scale – the lighter scale arising from the fine-tuning of  $\eta_0$  for *doublet-triplet splitting* alluded to earlier. To start with we consider  $\mu_i$  to be kept at the same order by a similar tuning of  $\eta_i$ , which according to Eqn. (2.8) keeps  $\mathcal{M}_i$  at the GUT scale<sup>3</sup>.

This matrix is diagonalised by going to the mass basis:

$$\begin{pmatrix} \tilde{H}_d^- \\ e_i \end{pmatrix} = \begin{pmatrix} \tilde{U}_{00} & \tilde{U}_{0j} \\ \tilde{U}_{i0} & \tilde{U}_{ij} \end{pmatrix} \begin{pmatrix} \tilde{H}_{0d}^- \\ e_{0j} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \tilde{H}_u^+ \\ e_i^c \end{pmatrix} = \begin{pmatrix} \tilde{V}_{00} & \tilde{V}_{0j} \\ \tilde{V}_{i0} & \tilde{V}_{ij} \end{pmatrix} \begin{pmatrix} \tilde{H}_{0u}^+ \\ e_{0j}^c \end{pmatrix}. \quad (2.17)$$

The matrix  $\tilde{U}$  is obtained by the replacement  $x_i \rightarrow y_i = \mu_i/\mu_0$  in Eqns. (2.12) - (2.14) and  $\tilde{V} \simeq \mathcal{I}$  so long as  $\mu_i \sim \mu_0 \gg m_{d_i}$ .

The mixing in the neutralino - neutrino sector – both Majorana fields – is well studied and, as is well known, leads to one neutrino state getting a non-zero mass. We return to this later.

The message from this analysis is that an extension of bilinear  $R$ -parity violation to  $SU(5)$  leads to mixing between the colour anti-triplets  $\bar{T}$  and  $d_i^c$  – Eqn. (2.10) – besides the much studied  $\tilde{H}_d^- - e_i$  mixing given in Eqn. (2.17) and a similar mixing in the neutrino - neutralino sector. The natural magnitude of these mixings is  $\mathcal{O}(1)$ . As for the usual doublet-triplet mixing, it is possible through fine-tuning to make one of these, but not both, to be small.

### III. RPV SUSY $SU(5)$ : TRILINEAR COUPLINGS

The Yukawa couplings for  $\bar{T}$  arise from the  $Y^5$  term in Eqn. (2.5) which can be written as

$$Y_i^5 \left[ \bar{T}_\rho \{ \epsilon_{\rho\xi\sigma} d_{i\xi}^c u_{i\sigma}^c + (u_{i\rho} e_i - d_{i\rho} \nu_{e_i}) \} - H_d^- \{ d_{i\alpha}^c u_{i\alpha} + \nu_{e_i} e_i^c \} + H_d^0 \{ d_{i\alpha}^c d_{i\alpha} + e_i e_i^c \} \right], \quad (3.1)$$

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<sup>3</sup> We show later that the smallness of the neutrino mass calls for a far smaller  $\mu_i$ , i.e., a higher degree of fine-tuning.

where  $\rho, \xi, \sigma$  are colour indices.

When expressed in the mass basis using Eqns. (2.10) and (2.17), it generates the  $\lambda, \lambda'$  and  $\lambda''$  trilinear couplings in the following RPV superpotential

$$\mathcal{W}_{\mathcal{R}} = \frac{1}{2} \lambda_{ijk} L_i L_j e_k^c + \lambda'_{ijk} L_i Q_{j\rho} d_{k\rho}^c + \frac{1}{2} \lambda''_{ijk} \epsilon^{\rho\xi\sigma} u_{i\rho}^c d_{j\xi}^c d_{k\sigma}^c + \mu_i L_i H_u, \quad (3.2)$$

where

$$\lambda'_{iik} = -Y_i^5 U_{k0}, \quad (3.3)$$

$$\lambda''_{ijk} = -Y_i^5 (U_{k0} U_{ij} - U_{j0} U_{ik}). \quad (3.4)$$

Here we neglect rotation due to  $V$  and, as noted earlier, work in a basis where the matrix  $Y_{0ij}^5$  is diagonal with the elements proportional to the down-type quark or charged lepton masses. We note that not all the  $\lambda'$  couplings are obtained.  $\lambda$ -type couplings are generated due to the bilinear mixing terms  $\mu_i$ . These mixings also modify Eqns. (3.3 - 3.4). As the  $\mu_i$  turn out to be rather small, we do not display these effects here. RPV originally contained in  $\mathcal{M}_i$  is now manifested in the form of trilinear RPV couplings as above.

#### IV. NEUTRINO MASS AND PROTON DECAY CONSTRAINTS

As mentioned earlier, the presence of non-zero  $\mu_i$  leads to neutrino-neutralino mixing and the neutralino mass matrix is extended to incorporate the massless neutrinos. During diagonalisation of this matrix the small  $\mu_i$  terms along with the much heavier neutralino masses lead to a seesaw-like contribution to neutrino masses. This way only one neutrino (can be chosen as the heaviest one) becomes massive whereas the other neutrinos can get loop level contributions in the RPV scenario. A rough estimate yields  $m_\nu \sim \sum_i \mu_i^2 / \tilde{m}$  [1, 2], where  $\tilde{m}$  represents neutralino mass and is set to  $\mu_0 \sim 1$  TeV. To reproduce  $m_\nu \sim 0.1$  eV, one requires  $\mu_i \sim 10^{-3}$  GeV, i.e.  $y_i = (\mu_i / \mu_0) \sim 10^{-6}$ . Due to this hierarchy between  $\mu_0$  and  $\mu_i$ , when the charged fermion mass matrix in (2.16) is diagonalised,  $m^{\text{diag}} \sim \text{diag}(m_{\ell_i})$  implying  $Y_i^5 \simeq m_{\ell_i} / v_d$  in a basis in which  $Y_{0ij}^5$  is diagonal.

In the presence of both  $\lambda'$  and  $\lambda''$  couplings, proton decay can proceed through  $p \rightarrow e^+ + \pi^0$  and  $p \rightarrow \bar{\nu}_e + K^+$  as shown in Fig. 1. The SuperKamiokande collaboration has put lower limits [7, 8] on the proton lifetime for decays *via*  $p \rightarrow e^+ + \pi^0$  and  $p \rightarrow \nu_e + K^+$  channels



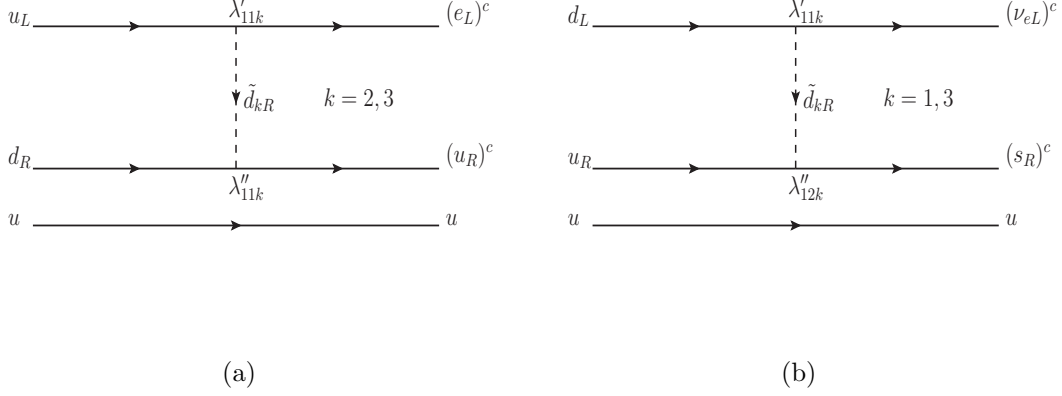


FIG. 1. Proton decay channels (a)  $p \rightarrow e^+ + \pi^0$  and (b)  $p \rightarrow \bar{\nu}_e + K^+$  mediated by trilinear RPV couplings.

as  $8.2 \times 10^{33}$  years and  $5.9 \times 10^{33}$  years respectively. These limits constrain the strength of RPV [16] and we find:

$$|\lambda'_{11k} \lambda''_{11k}| < 2.0 \times 10^{-25} \quad \text{and} \quad |\lambda'_{11k} \lambda''_{12k}| < 1.6 \times 10^{-25}, \quad (4.1)$$

for squark mass  $\tilde{m} = 1$  TeV.

Using Eqn. (3.3), the above constraints can be used to write

$$|(Y_1^5)^2 U_{k0}(U_{k0} U_{11} - U_{10} U_{1k})| < 2.0 \times 10^{-25}, \quad k = 2, 3, \quad (4.2)$$

$$|(Y_1^5)^2 U_{k0}(U_{20} U_{1k} - U_{k0} U_{12})| < 1.6 \times 10^{-25}, \quad k = 1, 3. \quad (4.3)$$

Now using Eqn. (2.12 - 2.14), one can restrict the bilinear RPV terms involving  $\bar{T}$  as  $x_1 < 1.7 \times 10^{-7}$ ,  $x_2 < 2.1 \times 10^{-7}$  and  $x_3 < 2.1 \times 10^{-7}$ .

Thus the smallness of the neutrino masses and the stringent proton decay lifetimes demand that  $x_i, y_i \ll 1$ . It is impossible to accomplish this from Eqns. (2.7) and (2.8) by any choice of  $\eta_i$ . So extension of bilinear RPV to SUSY SU(5) has a serious difficulty.

One way to bypass this conundrum is to keep the trilinear RPV terms,  $Y_{ijk}^5$ , in Eqn. (2.5) to be non-zero and make additional fine-tunings so that they almost exactly cancel off a large contribution emerging from the  $x_i$  leaving a small remainder consistent with proton decay and further ensure through other fine-tunings that the neutrino mass remains small enough [3]. Alternatively, one may abandon the principle of naturalness and set  $\eta_i = 0$  so

that  $\mathcal{M}_i = \mu_i = M_i$  and choose  $M_i$  satisfying the bound on  $y_i$  from the neutrino mass<sup>4</sup>. The limits on  $x_i$  then imply that the matrix  $U$  like  $V$  is almost an unit matrix. Hence,  $m^{\text{diag}} \approx m_d = \text{diag}(m_{d_i})$ . However,  $m^{\text{diag}} \approx \text{diag}(m_{\ell_i})$  from discussions following Eqn. (2.16). So finally,  $m_{d_i} \approx m_{\ell_i}$ , or in other words, bilinear RPV couplings  $\mathcal{M}_i$  and  $\mu_i$  do not help correct the wrong fermion mass ratio problem posed by RPC SUSY SU(5) scenario.

## V. RPV CANNOT SOLVE THE COLOUR TRIPLET MASS PROBLEM

As MSSM is promoted to SUSY SU(5), RG running of the gauge coupling constants are affected by the mass of the colour triplets  $T$  and  $\bar{T}$ . Thus, the requirement of gauge coupling unification relates the GUT scale with  $M_T$ . We can roughly see at 90% CL [17],

$$3.5 \times 10^{14} < M_T \text{ (GeV)} < 3.6 \times 10^{15}, \quad (5.1)$$

whereas the proton decay constraints puts a lower bound on  $M_T$ :

$$M_T > 7.6 \times 10^{16} \text{ GeV}. \quad (5.2)$$

It leads to a discrepancy in the minimal SUSY SU(5) framework. It is interesting to explore whether RPV can at least provide a solution to this problem.

In [18] the changes in the grand unification scale due to the presence of RPV couplings have been explored. It has been shown for order one RPV couplings at the electroweak scale the change in  $M_{GUT}$  is at the most 20%. Hence, it cannot solve the disparity in the above two bounds, unless the RPV contribution to proton decay destructively interferes in a fine-tuned manner with colour triplet mediated proton decay.

In the situation discussed here, namely, that the sole RPV is generated from the  $\mathcal{M}_i$  terms, the  $\lambda'$  and  $\lambda''$  couplings so produced are constrained from proton decay to be way too small to make any appreciable effect on gauge coupling unification to address the above disparity.

## VI. SUMMARY AND CONCLUSION

Promoting  $R$ -parity violation from the MSSM to SUSY SU(5) introduces a new type of bilinear RPV coupling  $\mathcal{M}_i$  involving the down-type antiquarks of the matter  $\bar{5}$ -plets and the

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<sup>4</sup>  $\eta_0 \neq 0$  produces doublet-triplet splitting, i.e.,  $\mathcal{M}_0 \gg \mu_0$ . So, now  $M_i/\mathcal{M}_0 = x_i \ll y_i = M_i/\mu_0$ .

superpartner of SU(2) singlet, colour anti-triplets  $\bar{T}$  which are members of the SU(5) Higgs  $\bar{5}$ -plet. This is in addition to the usual bilinear RPV terms,  $\mu_i$ , that induce mixing between the SU(2) doublet leptons and the Higgsino.

The mixing among the colour anti-triplet states resulting from the diagonalisation of the mass matrix introduces trilinear RPV  $\lambda'$  and  $\lambda''$  terms which can lead to proton decay. The strong bounds ensuing from non-observation of proton decay in experiments such as SuperKamiokande restrict these RPV couplings to  $\mathcal{M}_i/\mathcal{M}_0 \sim 10^{-7}$ . On the other hand the  $\mu_i$  terms result in neutrino-neutralino mixing at the tree level. The smallness of the neutrino masses *vis-à-vis* the weak scale implies that the ratio  $\mu_i/\mu_0$  are also quite suppressed –  $\sim 10^{-6}$ .

The type of fine-tuning that introduces doublet-triplet mass splitting within the SU(5) Higgs  $\bar{5}$ -plet can be extended to the RPV sector. This can suppress either the  $\bar{T} - d^c$  mixing or the standard bilinear RPV couplings involving leptons, but not both simultaneously. This is an obstacle to extending RPV to SUSY SU(5).

One way to circumvent this impasse is to assume further fine-tuned cancellations across different sectors by (a) introducing trilinear RPV terms in the Lagrangian which precisely compensate the ones generated through the mechanism above to leave a small remnant that is consistent with proton decay limits, and (b) ensure through a different set of fine-tunings that the tree- and loop-level contributions to the neutrino mass remain under control.

One may instead not abide by the principle of naturalness, take  $\eta_i = 0$ , and choose  $M_i$  so that  $y_i = M_i/\mu_0 \sim 10^{-6}$ . Then  $x_i = M_i/\mathcal{M}_0 \sim 10^{-20}$  is utterly negligible. SU(5) symmetry dictates  $m_{d_i} = m_{\ell_i}$  as both down-type quark and charged lepton Yukawa couplings originate from the  $Y^5$  term in the SUSY SU(5) superpotential in Eqn. (2.5). Due to the presence of  $\mathcal{M}_i$  and  $\mu_i$ , it may appear that the mass ratios  $m_{d_i}/m_{\ell_i}$  can be altered as desired by adjusting these terms. However, because of the tight constraint on  $x_i$  the change in the down-type quark masses are not appreciable. The weak scale masses in  $m^{\text{diag}}$  in Eqn. (2.9) also induce a small mixing  $\sim \mathcal{O}(M_W/M_{GUT})$  between the left-handed down-type quarks with  $T$ . The effect of such small mixings on the down-type quark masses is insignificant. The usual bilinear RPV terms,  $\mu_i$ , tightly constrained by the size of the neutrino mass, induce mixing between the charged leptons and the charged Higgsino. This also leads to a modification in the charged lepton masses but it is negligibly small.

In summary, promoting bilinear RPV to SUSY SU(5) faces a naturalness obstacle from the twin limits on proton decay lifetime and the neutrino mass. Extending the fine-tuning that ensures doublet-triplet splitting in the Higgs multiplet to SUSY does not provide a solution. One way out is to abandon the naturalness principle itself. An alternate possibility is to invoke several further fine-tunings in sectors which are *a priori* unrelated. Without such a procedure one cannot change the SU(5) prediction of the mass ratio  $m_{d_i}/m_{\ell_i}$  significantly.

## VII. ACKNOWLEDGEMENTS

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### Appendix A: The mixing matrix

In case only one generation of fermions is considered, Eqn. (2.9) looks like

$$\begin{pmatrix} \bar{T}_0 & d_0^c \end{pmatrix} \begin{pmatrix} \mathcal{M}_0 & 0 \\ \mathcal{M}_1 & m_1 \end{pmatrix} \begin{pmatrix} T_0 \\ d_0 \end{pmatrix} \equiv \begin{pmatrix} \bar{T}_0 & d_0^c \end{pmatrix} \mathcal{M} \begin{pmatrix} T_0 \\ d_0 \end{pmatrix}, \quad (\text{A1})$$

where  $m_1 = Y_1^5 v_d$ .

The mass matrix  $\mathcal{M}$  can be diagonalised by a bi-unitary transformation:

$$\begin{pmatrix} \bar{T} & d^c \end{pmatrix} \begin{pmatrix} M_T & 0 \\ 0 & m_d \end{pmatrix} \begin{pmatrix} T \\ d \end{pmatrix} = \begin{pmatrix} \bar{T} & d^c \end{pmatrix} [U_R \mathcal{M} U_L^\dagger] \begin{pmatrix} T \\ d \end{pmatrix}, \quad (\text{A2})$$

where  $U_R \mathcal{M} \mathcal{M}^\dagger U_R^\dagger = U_L \mathcal{M}^\dagger \mathcal{M} U_L^\dagger = \text{diag}\{M_T^2, m_d^2\}$  and the mass basis eigenstates are:

$$\begin{aligned} \begin{pmatrix} \bar{T} \\ d^c \end{pmatrix} &= U_R \begin{pmatrix} \bar{T}_0 \\ d_0^c \end{pmatrix} = \begin{pmatrix} c_R & s_R \\ -s_R & c_R \end{pmatrix} \begin{pmatrix} \bar{T}_0 \\ d_0^c \end{pmatrix}, \\ \begin{pmatrix} T \\ d \end{pmatrix} &= U_L \begin{pmatrix} T_0 \\ d_0 \end{pmatrix} = \begin{pmatrix} c_L & s_L \\ -s_L & c_L \end{pmatrix} \begin{pmatrix} T_0 \\ d_0 \end{pmatrix}. \end{aligned} \quad (\text{A3})$$

Above,  $c_{L,R} = \cos \theta_{L,R}$ ,  $s_{L,R} = \sin \theta_{L,R}$ . The mass eigenvalues (ignoring terms of  $\mathcal{O}(m_1^4)$ ) are:

$$M_T = \mathcal{M}_0 \sqrt{1 + x^2 + \frac{x^2 z^2}{1 + x^2}} \simeq \mathcal{M}_0 \sqrt{1 + x^2}, \quad m_d = \mathcal{M}_0 \sqrt{\frac{z^2}{1 + x^2}} \simeq 0, \quad (\text{A4})$$

where  $x = \mathcal{M}_1/\mathcal{M}_0$  and  $z = m_1/\mathcal{M}_0 \ll x$  since  $m_1 \sim \mathcal{O}(M_W)$  and  $\mathcal{M}_0 \sim \mathcal{M}_1 \sim \mathcal{O}(M_{GUT})$ . The mixing angles are given by:

$$\tan 2\theta_R = \frac{2x}{1-x^2-z^2} \simeq \frac{2x}{1-x^2} \quad \text{and} \quad \tan 2\theta_L = \frac{2zx}{1+x^2-z^2} \simeq 0. \quad (\text{A5})$$

Thus, the mixing between  $T$  and  $d$  is negligible while that between  $\bar{T}$  and  $d^c$  can be significant. Indeed, as  $x \rightarrow 1$  the mixing angle  $\theta_R$  tends to its maximal value of  $\pi/4$ . On the other hand for the  $x \rightarrow 0$  limit, as expected  $\theta_R \rightarrow 0$ .

From Eqn. (A5) to a good approximation:

$$\cos \theta_R = \frac{1}{\sqrt{1+x^2}}, \quad \sin \theta_R = \frac{x}{\sqrt{1+x^2}}, \quad \cos \theta_L = 1, \quad \sin \theta_L = 0. \quad (\text{A6})$$

This result can be readily extended to three generations.

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